A TECHNIQUE FOR A POSTERIORI ESTIMATION OF WAVE REFLECTIONS

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A numerical treatment of wave propagation in unbounded domains requires the use of an absorbing boundary condition (ABC) to take into account the radiation damping contributed by the unbounded domain. With an ABC, one has only to discretize a finite computational domain bounded by an artificial boundary. This class of problems pose a tremendous computational challenge in that global ABC's, which are very accurate, tend to be extremely expensive to determine, while local ABC's, which are much more efficient, can give erroneous results [1]. A desirable strategy would then be to use an adaptive scheme, in which an absorbing boundary condition of an optimal order of accuracy is used to control, within a prescribed tolerance, the amount of spurious wave reflections from the artificial boundary. Such an adaptive scheme should be equipped with two techniques: the first, an absorbing boundary condition of arbitrary high-order, and the second, a technique for *a posteriori* estimation of wave reflections from the artificial boundary.

The present study addresses the second ingredient of an adaptive strategy in the simplest possible context: one-dimensional wave propagation in a homogeneous and elastic bar. In this case, the energy flux defined by

$$I_x \equiv -E\frac{\partial u}{\partial x}\frac{\partial u}{\partial t}, \tag{1}$$

and evaluated in the vicinity of the artificial boundary can provide the magnitude and direction of energy propagation due to wave reflections, where u is the axial displacement, and E the Young's modulus. However, I_x based on the computed solution contains not only the energy flux due to reflected waves, but also the flux from the interior. That is, the reflected flux may be overridden by the flux from the interior, thus giving the false information that all the waves are outgoing even if there are spurious reflections.

We propose to solve the wave equation locally, on a single space-time element, using the discontinuous Galerkin method [1]. The concept of the domain of determinacy is employed for the local solution procedure, in which the influence of the energy flux from the interior is excluded. The value of I_x obtained by this procedure then includes only the effects of reflected waves. A numerical experiment demonstrates this “filtering action” of the designed scheme in extracting the energy flux due to reflections only. A simple adaptive scheme is implemented based on the a posteriori estimation technique developed.

References

[1] S.-H. Park and J.L. Tassoulas, “A discontinuous Galerkin method for transient analysis of wave propagation in unbounded domains,” *Computer Methods in Applied Mechanics and Engineering*, v. 191, p. 3983-4011, 2002.